I can prove that GMRES converges to the exact solution in two steps for the given matrix A and any right-hand side vector b, regardless of the initial guess x0. Here's the explanation:

**Key to the Proof: Upper Triangular Matrix**

The critical aspect of this proof lies in the specific structure of matrix A. It's an upper triangular matrix, meaning all elements below the main diagonal are zero.

A = [[1, 0, a],

[0, 1, b],

[0, 0, 1]]

This property plays a crucial role in how GMRES builds the Krylov subspace, which is a sequence of vectors used to approximate the solution.

**GMRES and Krylov Subspace**

1. **Initial Guess and Residual:** GMRES starts with the initial guess x0 and calculates the initial residual r0 = b - A \* x0.
2. **Krylov Subspace Generation:** The Krylov subspace K(A, r0) is constructed based on r0, which includes vectors generated by multiplying A repeatedly with r0. However, since A is upper triangular, these multiplications will only introduce zeros below the diagonal, preserving the upper triangular structure in subsequent vectors.

**Crucial Step: Spanning the Null Space**

Because of the upper triangular nature of A and the generated vectors in the Krylov subspace, the second step (k = 1) of GMRES will already span the entire null space of A. The null space refers to all vectors x that satisfy A \* x = 0.

**Why is this important?**

In solving Ax = b, we essentially want to find a solution x that makes the residual (b - Ax) as close to zero as possible. Since the null space of A represents solutions that make Ax zero, having the Krylov subspace span this null space in the second step guarantees that GMRES can find a solution that reduces the residual to zero.

**Reaching the Exact Solution in Two Steps**

Therefore, after two steps of GMRES:

1. The initial residual r0 is addressed in the first step.
2. The second step, due to the upper triangular structure, ensures the Krylov subspace encompasses the entire null space, allowing GMRES to find a solution that drives the residual to zero.

This effectively achieves the exact solution in two steps for any right-hand side vector b and any initial guess x0.

**Note:** This property of GMRES converging in two steps holds true specifically for upper triangular matrices. In general, GMRES might require more iterations for different matrix structures.